Comparing the results with those of other researchers in the field of heat transfer in triangular ducts, it should be borne in mind, that the bulk of the references deal with tests in long ducts with fully, or nearly fully developed flow and in all cases, either uniform wall temperature or uniform heat flux. In the present case only a short triangular passage was used and the heating was only at the two ends of the fin. Furthermore, the value for the Nusselt number expressed in the equation (2), does not apply to the whole passage, but only to the surface of the fin.

It should be pointed out here, that the value of Nusselt number defined by equation (2) can at this stage be accepted as valid only for a fin in an array of triangular passages with apex angle of 36·9°. Any change of the geometry of the triangular passage may change the flow and film coefficient distribution along the wall and consequently the value of mean film coefficient.

Further testing would have to be carried out on a range of apex angles and fin geometries to establish a more universal validity.

CONCLUSIONS

1. The temperature distribution on the surface is not symmetrical and the position of minimum temperature on the surface is independent of the Reynolds number.
2. The temperature distribution on the two surfaces are identical, but are of mirror image form, thereby creating a displacement between the positions of minimum temperatures on the two surfaces.
3. The value of the average Nusselt number for the fin has been found to be $Nu = C Re^{0.8}$, the value of the constant being $C = 0{\cdot}01085$. The validity of this Nusselt number correlation for different geometries requires further investigation.

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A TWO-DIMENSIONAL ANALOG SIMULATING THE HELMHOLTZ EQUATION FOR HEAT FLOW

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NOMENCLATURE

A_f, fin surface area [m^2];
A_m, mean area between *iso*-potentials [m^2];
C, a constant;
E, electrostatic field strength [V/m^2];
h, heat-transfer coefficient [W/m^{2}K];
i_z, current density in the z direction [A/m^2];
I, current [A];
k, thermal conductivity [W/mK];
P, constant voltage potential [V];
q, flow;
r, radius [m];
S, current density [A/m^2];
t, paper thickness [m];
V, voltage potential [V];
w, characteristic fin length [m].

Greek symbols

γ, electrical conductivity [Ω^{-1}/m];
δ, fin thickness [m];
δ_l, laminate thickness [m];
θ, potential;
ϕ, fin efficiency;
λ, a constant;
μ, a constant;
∇^2, two-dimensional Laplacian operator $= \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$.

Subscripts

0, fin root;
m, mean;
x, y, z, refers to directions of Cartesian co-ordinate axes.

INTRODUCTION

In two-dimensional Cartesian co-ordinates, a Laplacian potential field in the xOy plane is represented by

$$\nabla^2\theta(x,y) = 0. \tag{1}$$

If this field of $\theta(x,y)$ is constrained to permit a flow, $q(x,y)$, normal to the xOy plane, such that it is directly proportional to the local potential, it can be shown that the field will be distorted into one represented by the Helmholtz equation.

$$\nabla^2\theta - \lambda^2\theta = 0 \tag{2}$$

where λ^2 = a constant.

Electrical conducting paper analogies are widely used in the simulation of Laplacian fields [1]. A technique to distort such a potential field into one represented by equation (2), has been developed.

THE ELECTRICAL FIELD IN A SHEET OF CONDUCTION PAPER

Consider a sheet of homogeneous and isotropic conducting paper having uniform electrical properties. If the plane of the paper represents the xOy plane, then, when the paper is subjected to a steady current flow,

$$S = \gamma E \tag{3}$$

where

$$E = -\operatorname{grad} V(x,y) \tag{4}$$

$$\therefore \quad S = -\gamma \operatorname{grad} V(x,y). \tag{5}$$

Assuming the paper has a finite but negligible thickness,

$$\partial V/\partial z = 0; \quad S_z = 0.$$

For flow continuity,

$$\operatorname{div} S = \operatorname{div}(-\gamma \operatorname{grad} V) = 0 \tag{6}$$

where now

$$\operatorname{div} S = \frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y}. \tag{7}$$

If the electrical conductivity, γ, is constant,

$$\nabla^2 V(x,y) = 0. \tag{8}$$

Now consider this Laplacian field with a local current density, $i_z(x,y)$, externally induced over the paper, normal to the xOy plane. The flow continuity equations now give

$$\operatorname{div} S = -\frac{i_z(x,y)}{t(x,y)} = -\gamma \operatorname{div}\operatorname{grad} V(x,y). \tag{9}$$

If i_z is proportional to the local voltage potential, $V(x,y)$, and if the thickness, $t(x,y)$, is constant

$$\nabla^2 V - \mu^2 V = 0 \tag{10}$$

where μ^2 is a constant dependent upon the physical and electrical characteristics of the paper.

THE ANALOG

If a lamination is made up from sheets of conducting paper pasted together, and if in addition to potentials applied to boundaries on the top surface, a portion of the bottom surface is held at a potential P, a normal current flow will be induced over this region. The local current density will be given by

$$i_z(x,y) = \frac{\gamma_z(x,y)}{\delta_l(x,y)}[V(x,y) - P]. \tag{11}$$

Hence

$$\nabla^2[V-P] - (\gamma_z/\gamma\delta_l^2)[V-P] = 0 \tag{12}$$

If γ, γ_z, δ_l, and P, are all constant, equation (12) will be directly analogous to equations (2) and (10). Under these conditions,

$$[V-P] \equiv 0; \quad i_z \equiv q; \quad (\gamma_z/\gamma\delta_l^2) = \mu^2 \equiv \lambda^2.$$

The energy equations applied to a constant thickness fin, surrounding a round tube yield

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{2h}{k\delta}(T - T_s). \tag{13}$$

Equation (13) is based on the following assumptions:

1. Heat flow is steady.
2. The fin material is homogeneous and isotropic.
3. The thermal conductivity, k, of the material remains constant.
4. The temperature at the fin root, T_0, remains constant.

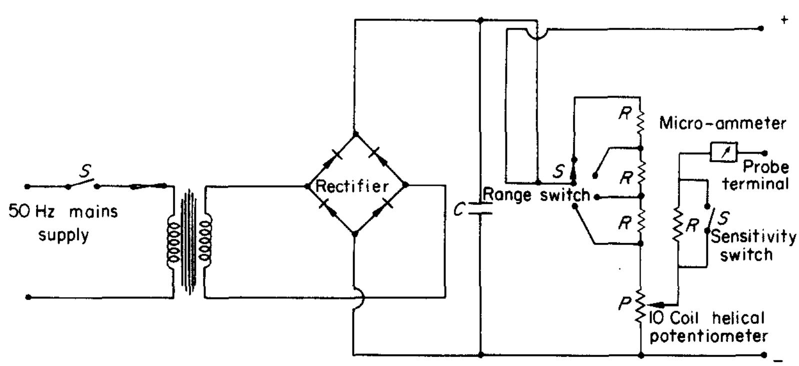

Fig. 1. A typical field plotter circuit.

5. The temperature of the surrounding medium, T_s, remains constant.
6. The heat-transfer coefficient, h, is uniform.
7. The fin thickness, δ, is small.
8. There is no heat flow through the fin edge.

In terms of the thermal potential, θ,

$$\nabla^2\theta - \lambda^2\theta = 0 \tag{14}$$

where

$$\lambda^2 = 2h/k\delta.$$

For annular fins, equation (13) is a form of the modified Bessel equation, and Gardner [2] has presented solutions for such fins. Defining finning efficiency, ϕ, as

$$\frac{\text{Heat dissipated by the fin}}{\text{Heat dissipation if entire fin surface was at root temperature}}$$

$$\phi = \int_0^{A_f} \theta \, dA/\theta_0 A_f. \tag{15}$$

Since $\phi = f(\theta)$, the solution for θ may be given instead, in terms of the efficiency, ϕ. Gardner presents the efficiency as a function of w, r_0, and λ; where w is a characteristic fin dimension and r_0 is the fin root radius.

Such analytic solutions are not possible for square and rectangular fins, but approximate numerical solutions [3–5] are available.

THE MODELLING TECHNIQUE

Laminations made up from sheets of graphitised (Teledeltos) paper were tested to determine their electrical properties. Though single sheets of paper had a resistance of 1500 ± 3 per cent Ω per square along both axes, some variation in the values of γ, and γz, was observed for the laminations. Qualitatively, this was consistent with the observed variation in the thicknesses of adhesive layers used. It is evident though, that if the parameter $(\gamma_z/\gamma\delta_l^2)$ remains constant, the Helmholtz equation will still apply. Direct measurement of the value of $(\gamma_z/\gamma\delta_l^2)$ proved difficult and the simulated value of λ^2 was derived instead.

A number of models representing two-dimensional, constant thickness, annular fins were constructed from a single sample of laminate. The required boundary conditions were simulated exactly, by subjecting different regions on the model to appropriate voltage potentials.

Using a field plotter (Alpha PR, Fig. 1) *iso*-potentials were mapped over the model surface. The efficiencies of the simulated fins were then calculated from a numerical form of equation (15):

$$\phi = \sum_0^{A_f} \theta_m A_m/\theta_0 A_f \tag{16}$$

where A_m = area of model surface between *iso*-potentials and θ_m = mean potential applicable to A_m. These values of efficiency were fitted to curves representing Gardner's solution [2]. The fit obtained (Fig. 2) indicated that the simulated value of λ^2, and hence of $(\gamma_z/\gamma\delta_l^2)$ varied by no more than ± 5 per cent. This was reproduced with other samples, confirming that the parameter $(\gamma_z/\gamma\delta_l^2)$ could effectively be considered a constant over any particular sample of laminate.

This technique was verified by the simulation of square and rectangular fins having known numerical solutions. Various samples of laminate were made, and from each sample three models of constant thickness annular fins were constructed. The values of λ^2 being simulated were con-

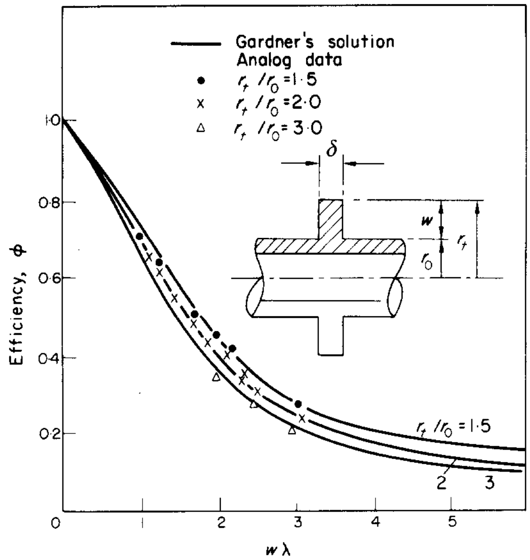

FIG. 2. Efficiency of constant thickness annular fins.

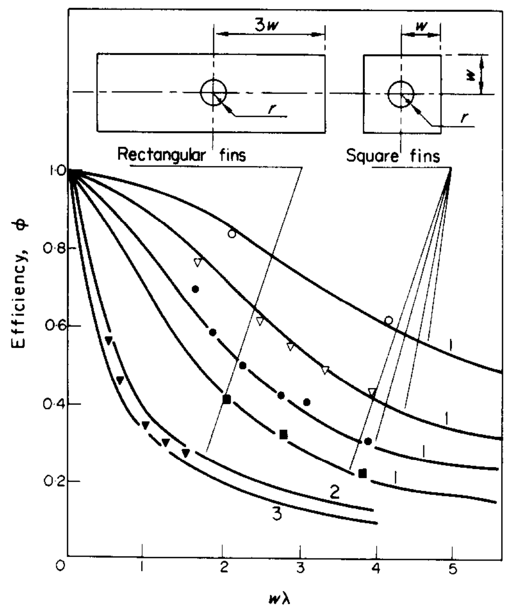

FIG. 3. Efficiency of square and rectangular fins.

Approximate solutions	Analog data
1, [3]	○, $w/r = 1.2$
2, [3]	▽, $w/r = 1.5$
3, [5]	●, $w/r = 2.0$
	■, $w/r = 3.0$
	▼, $w/r = 2.0$

sequently quantified by calibration against Gardner's curves. Table 1 shows these values of simulated λ for various samples of laminate. These values were then used in the simulation of square and rectangular fins. Derived values of fin efficiency are shown compared with numerical results in Fig. 3.

Table 1. Derived values of λ for various samples of laminate

Sample No.	No. of paper thicknesses	Derived λ (m^{-1})	λ mean (m^{-1})
1	3	49·606	49·675
2	3	48·819	
3	3	49·600	
4	4	42·323	42·717
5	4	42·520	
6	4	43·307	
7	4	42·520	
8	4	42·913	
9	5	34·646	34·252
10	5	33·858	

CONCLUSIONS

The accuracy obtainable with this analog is determined by the error in the calibrated value of λ. This can, with care, be kept within 10 per cent, and by varying the number of paper thicknesses in the lamination, a wide range of λ may be simulated. A comparison of analytical and numerical data with analog results shows a deviation of less than ± 7 per cent, and this technique has been used successfully in the evaluation of sheet fins which have not been amenable to analytical or numerical solution.

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ON THE SOLUTION OF THE HEAT EQUATION WITH TIME DEPENDENT COEFFICIENT

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NOMENCLATURE

- $A(N,\tau)$, $B(N,\tau)$, $\varphi(N,\tau)$, time dependent boundary functions defined on S;
- $f_0(M)$, initial distribution in V;
- $k(M,\tau)$, $w(M,\tau)$, $P(M,\tau)$, $\rho(M,\tau)$, prescribed functions defined in V;
- M, point in V;
- N, point on S;
- n, outward normal of S;
- $T(M,\tau)$, unsteady potential distribution defined in equation (1), (2) and (3);
- $\beta(\tau)$, $\gamma(\tau)$, prescribed function defined in τ;
- τ, time variable;
- $\psi_i(M,\tau)$, eigenfunctions;
- $\mu_i(\tau)$, eigenvalues.

In a recent paper [1], the author presented an analytical solution for a large class of heat-transfer problems. The present communication complements [1] applying the method of finite integral transforms for the solution of a more general mathematical model of transfer process with time and space dependent parameters.

Consider the following boundary value problem in a finite homogeneous region of arbitrary geometry V

$$\gamma(\tau)w(M,\tau)\frac{\partial T(M,\tau)}{\partial \tau} = \operatorname{div}[k(M,\tau)\operatorname{grad}T(M,\tau)] + [\beta(\tau)w(M,\tau)-\rho(M,\tau)]T(M,\tau)+P(M,\tau), \quad M\in V,\quad \tau\geqslant 0 \quad (1)$$

subject to the initial condition

$$T(M,0) = f_0(M) \quad (2)$$

and the boundary conditions

$$A(N,\tau)\frac{\partial T(N,\tau)}{\partial n} + B(N,\tau)T(N,\tau) = \varphi(N,\tau). \quad (3)$$

In [1] is solved the particular case where $w(M,\tau)$, $k(M,\tau)$, $\rho(M,\tau)$, $A(N,\tau)$ and $B(N,\tau)$ are not functions of the time τ.

It is supposed that the solution of the problem can be represented in the form of an eigenfunction expansion, with the assumption that the eingenvalue problem

$$\operatorname{div}[k(M,\tau)\operatorname{grad}\psi_i(M,\tau)] + [\mu_i^2(\tau)w(M,\tau)-\rho(M,\tau)]\psi_i(M,\tau) = 0 \quad (4)$$